Fundamentals of Engineering (FE) Examination
Electrical Engineering Section Review
August 31, 2009

Exam Format

The FE examination is 8 hours long, divided into two four-hour sessions. During the morning session, all examinees take a general exam consisting of 120 questions. The topics covered in the morning session are intended to be general and are drawn from twelve different areas, one of which is *Electricity and Magnetism*. In the morning session, approximately 9% of the questions relate to *Electricity and Magnetism*. During the afternoon session, examinees can opt to take a general exam or a discipline specific exam consisting of 60 questions.

The review material presented tonight is devoted primarily to the subjects covered in the morning session and general exam in the afternoon session.

References

Principle and Applications of Electrical Engineering, Giorgio Rizzoni, Fifth Edition, McGraw-Hill, 2007

Electrical Engineering Principles and Applications, Allan R. Hambley, Fourth Edition, Pearson/Prentice-Hall, 2008

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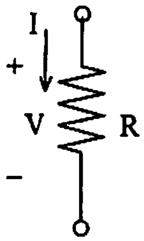
Electrical Engineering Session Reference Material

Fundamental Electrical Quantities

Q (Charge)	units: Coulombs	
I (Current)	units: Amperes	$i(t) = dq(t)/dt$
V (Voltage)	units: Volts	$v(t) = dw/dq$
W (Energy)	units: Joules	$w(t) = \int p(t)dt$
P (Power)	units: Watts	$p(t) = v(t)i(t) = dw(t)/dt$

Circuit Elements

Resistor: A passive circuit element that dissipates energy in the form of heat. The resistance of a resistor is expressed in ohms (Ω).



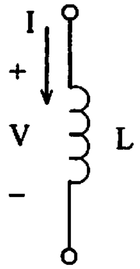
For a conductor of length L, electrical resistivity ρ, and area A, the resistance is: $R = \rho L/A$

Ohm's Law: $V = IR$; $v(t) = i(t)R$

Conductance $G = 1/R$
units: Siemens (S)

Power: $P = VI = I^2R = V^2/R$

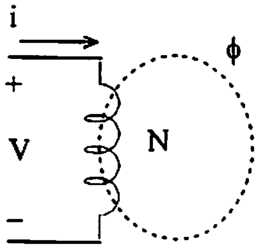
Inductor: An element that stores energy in a magnetic field. The inductance of an inductor is measured in Henries (H).



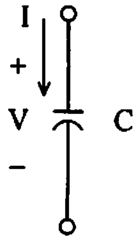
$L = N\phi/i$

Faraday's Law: $v(t) = L\, di_L/dt$

Energy Stored: $W_L(t) = \frac{1}{2}Li_L^2(t)$



Capacitor: An element that stores energy in an electric field. The capacitance of a capacitor is measured in Farads (F).

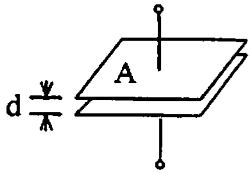


$C = \varepsilon A/d$

$q(t) = Cv_C(t)$; $i(t) = C\, dv_C/dt$

Energy Stored: $W_C(t) = \frac{1}{2}Cv_C^2(t)$

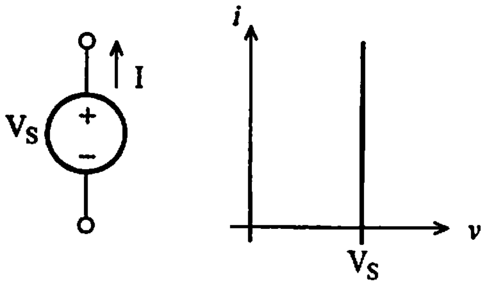
ε = dielectric constant

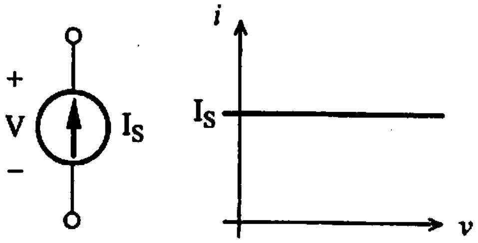


Energy Sources

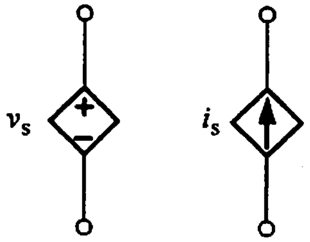
Ideal Voltage Source: This type of source supplies voltage with the polarity specified. The circuit that is connected determines the current that the source must supply.

Ideal Current Source: This type of source supplies current in the direction indicated. The circuit that is connected determines the voltage that appears across the current source.





Dependent (Controlled) Sources: The output of the source (voltage or current) is a function of some other voltage or current in a circuit.



Source type
Voltage controlled voltage source (VCVS)
Current controlled voltage source (CCVS)
Voltage controlled current source (VCCS)
Current controlled current source (CCCS)

Circuit Analysis Principles

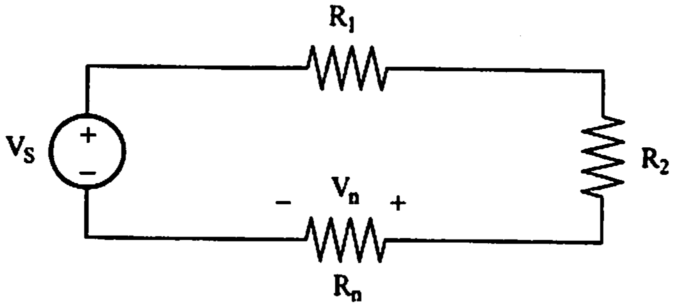
Kirchhoff's Laws: These two laws of conservation govern the behavior of all circuits.

Kirchhoff's Current Law (KCL): Conservation of charge, $\Sigma I = 0$ (at any node).

Kirchhoff's Voltage Law (KVL): Conservation of energy, $\Sigma V = 0$ (around any closed path).

Equivalent Resistance and Divider Principles:

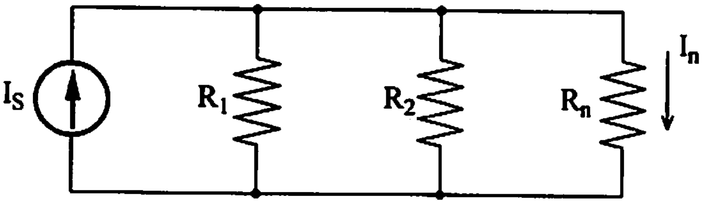
Series Circuits:



Series Resistors
$R_{EQ} = R_1 + R_2 + \ldots + R_n$

Voltage Division Principle
$V_n = V_S(R_n/R_{EQ})$

Parallel Circuits:



Parallel Resistors
$R_{EQ} = 1/(1/R_1 + 1/R_2 + \ldots + 1/R_n)$

Current Division Principle
$I_n = I_S(1/R_n/1/R_{EQ})$

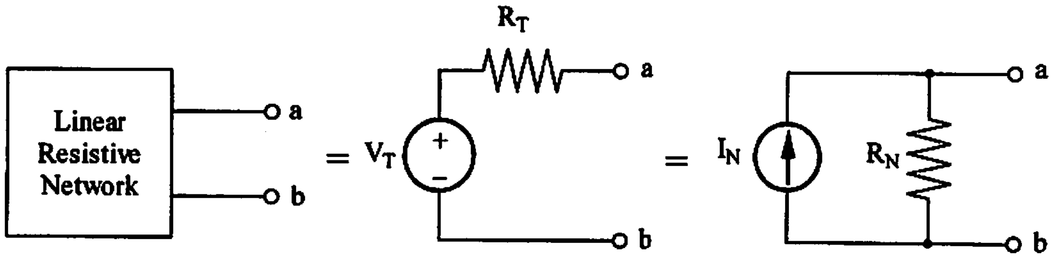
Node Voltage Analysis: Writing KCL equations at the independent nodes using Ohm's Law to express currents in terms of the voltages at the nodes ($I = \Delta V/R$).

Mesh Current Analysis: Writing KVL equations around the loops in a circuit using Ohm's Law to express voltages in terms of the mesh currents ($V = IR$).

Superposition: In any linear resistive network containing two or more independent sources, any circuit voltage or current may be calculated as the algebraic sum of all the individual voltages or currents caused by the sources acting alone. When zeroing sources voltage sources are replaced by short circuits and open circuits replace current sources.

Thévenin and Norton Equivalent Circuits: A simpler equivalent circuit consisting of a source and a resistor may replace any complex one-port network.

Complex Network — Thévenin Equivalent — Norton Equivalent



$V_T = V_{OC}$
$R_T = V_{OC}/I_{SC}$

$I_N = I_{SC}$
$R_N = V_{OC}/I_{SC}$

Maximum Power Transfer Theorem: A Thévenin equivalent circuit can be used to determine the load that will draw maximum power from a one-port network.

$$R_L = R_T \qquad P_{LMAX} = V_T^2/4R_T$$

RC and RL Transients: DC transients are caused when switching takes place in a circuit. The general form of the transient response for a first-order circuit with switched dc sources is described below.

For $t \geq 0$, $i(t) = I_\infty + (I_0 - I_\infty)e^{-t/\tau}$ $v(t) = V_\infty + (V_0 - V_\infty)e^{-t/\tau}$

Time Constant (τ): RL Circuit: $\tau = L/R$; RC Circuit: $\tau = RC$

Initial Values: (V_0 and I_0)
The voltage across a capacitor cannot change instantaneously ($v_{c0-} = v_{c0+}$).
The current through an inductor cannot change instantaneously ($i_{L0-} = i_{L0+}$).

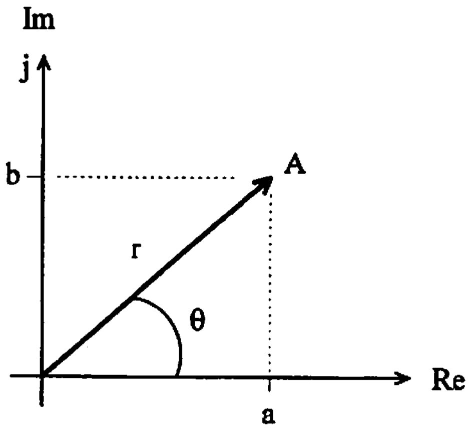
Final Values: (V_∞ and I_∞)
In dc steady state a capacitor looks like an open circuit and an inductor looks like a short circuit.

AC Circuits

Sinusoidal functions: $v(t) = Vm\cos(\omega t + \theta_V)$ Volts $i(t) = Im\cos(\omega t + \theta_I)$ Amps
Average value: $V_{AVG} = (1/T)\,{}_0\!\int^T v(t)\,dt$
Effective (RMS) value: $V_{RMS} = [(1/T)\,{}_0\!\int^T v^2(t)\,dt]^{1/2} = Vm/\sqrt{2}$
Sine-Cosine Relationships: $\sin(\omega t) = \cos(\omega t - 90°) = -\cos(\omega t + 90°)$

Frequency Domain Analysis: The use of phasors and impedance greatly simplifies the analysis of ac circuits. Once an ac circuit is represented in the frequency domain, all the same methods of circuit analysis developed for dc circuits apply. The only difference is that phasor and impedance quantities are complex numbers.

Complex Numbers:



Complex Number A

Rectangular Form: $A = a + jb$

Polar Form: $A = r\angle\theta$

Exponential Form: $A = re^{j\theta}$

R→P and P→R Conversions

$r = (a^2 + b^2)^{1/2}$; $\theta = \tan^{-1}(b/a)$

$a = r\cos\theta$; $b = r\sin\theta$

Phasors: $V = V_m e^{j\theta_V} = V_m\angle\theta_V$ $\quad I = I_m e^{j\theta_I} = I_m\angle\theta_I$ $\quad$ where $j = \sqrt{-1}$

Impedance: $Z = V/I = |Z|\angle\theta = R + jX$ (units: ohms(Ω))

Admittance: $Y = 1/Z$ (units: Siemens(S))

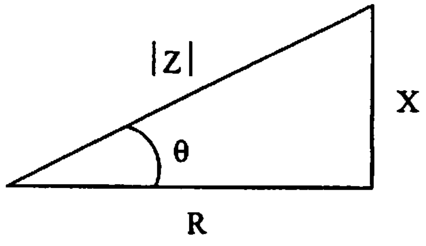
Circuit Element Impedances:

Resistor: $Z_R = R = R\angle 0°$

Inductor: $Z_L = j\omega L = \omega L\angle 90°$

Capacitor: $Z_C = 1/j\omega C = -j/\omega C = 1/\omega C\angle -90°$

Impedance Triangle:



Impedance $Z = |Z|\angle\theta = R + jX$

$R = \mathrm{Re}(Z)$: AC Resistance

$X = \mathrm{Im}(Z)$: AC Reactance

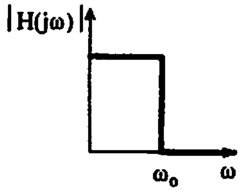
Capacitive Reactance: $X_C = -1/\omega C$

Inductive Reactance: $X_L = \omega L$

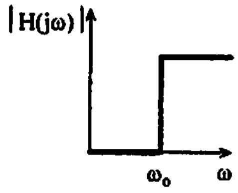
Frequency Response: Electrical filters pass components in a specific frequency range to the output without changing the amplitude or phase and reject or attenuate components at other frequencies. The transfer function of a filter is the phasor output divided by the phasor input. The transfer function is a complex quantity having both a magnitude response ($|H(j\omega)|$ and a phase response ($\angle H(j\omega)$).

$$H(j\omega) = V_{out}/V_{in} = |H(j\omega)|\ \angle H(j\omega)$$

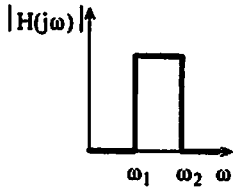
Low Pass Filter



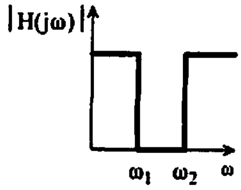
High Pass Filter



Band-pass Filter



Band-reject Filter



Resonance: For a circuit to be resonant it must have both types of energy storage elements. Resonant circuits are used to produce band-pass and band-reject filters.

Series Resonance:

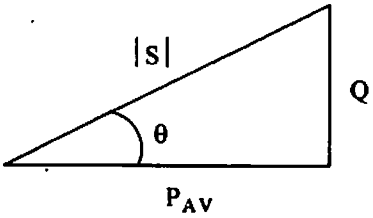
Resonant frequency: $\omega_n = 1/\sqrt{LC}$; $\omega_n L = 1/(\omega_n C)$; $Z_{EQ} = R$

Quality factor: $Q = \omega_n L/R = 1/(\omega_n CR)$

Bandwidth: $BW = \omega_n/Q$

AC Power

AC Complex Power: The expressions shown below are valid for rms quantities.



$S = VI^* = |S|\angle\theta = P_{AV} + jQ$

$|S| = VI = I^2|Z| = V^2/|Z|$ (Apparent Power (VA))

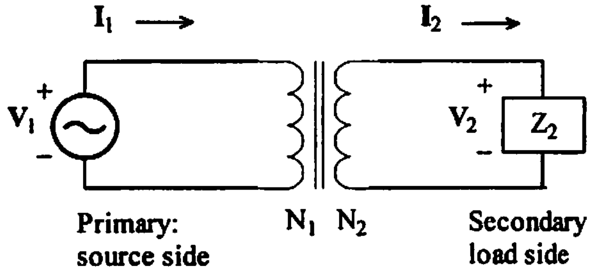
$P_{AV} = VI\cos\theta = I^2R = V_R^2/R$ (Average Power (W))

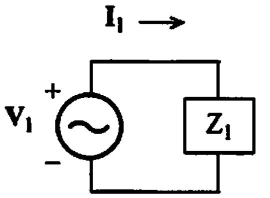
$Q = VI\sin\theta = I^2X = V_X^2/X$ (Reactive Power (VAR))

Power Factor (pf) $= \cos\theta = P_{AV}/|S|$

Inductive loads produce lagging power factors.
Capacitive loads produce leading power factors.

Ideal Transformers: A transformer is a magnetically coupled device. An ideal transformer has no losses.





Equivalent Circuit

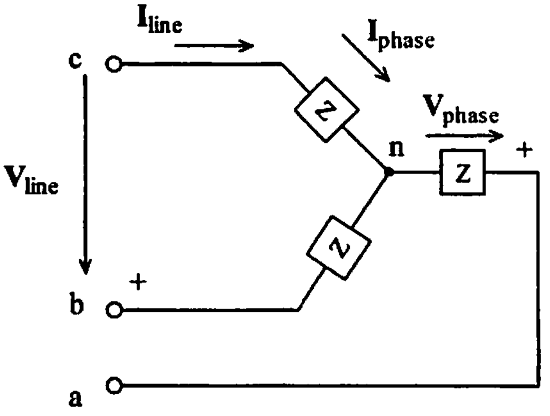
Voltage Transformation: $\frac{V_1}{V_2} = \frac{N_1}{N_2}$

Current Transformation: $\frac{I_1}{I_2} = \frac{N_2}{N_1}$

Impedance Transformation: $\frac{Z_1}{Z_2} = \left[\frac{N_1}{N_2}\right]^2$

Balanced three-phase systems: The information given below is in terms of rms quantities.

Balanced Wye (Y) System:



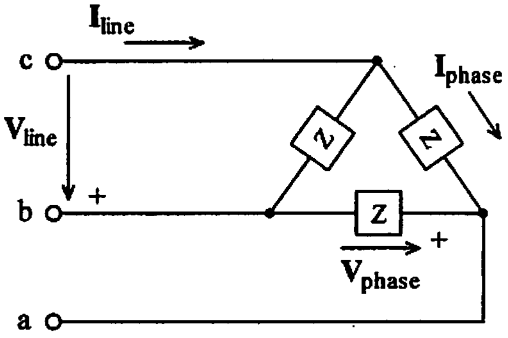
Balanced Wye Relationships:

$|V_{line}| = \sqrt{3}\,|V_{phase}|$; $\angle V_{line} = \angle V_{phase} + 30°$

$I_{line} = I_{phase}$

$P_{AV} = \sqrt{3}\, V_{line} I_{line} \cos\theta$; $Q = \sqrt{3}\, V_{line} I_{line} \sin\theta$; $|S| = \sqrt{3}\, V_{line} I_{line}$

Balanced Delta (Δ) System:



Balanced Delta Relationships:

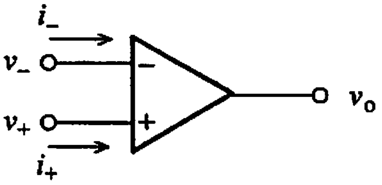
$|I_{line}| = \sqrt{3}\,|I_{phase}|$; $\angle I_{line} = \angle I_{phase} - 30°$

$V_{line} = V_{phase}$

$P_{AV} = \sqrt{3}\,V_{line}I_{line}\cos\theta$; $Q = \sqrt{3}\,V_{line}I_{line}\sin\theta$; $|S| = \sqrt{3}\,V_{line}I_{line}$

Operational Amplifiers

A very versatile Integrated Circuit (IC) that has many applications in electric circuits.

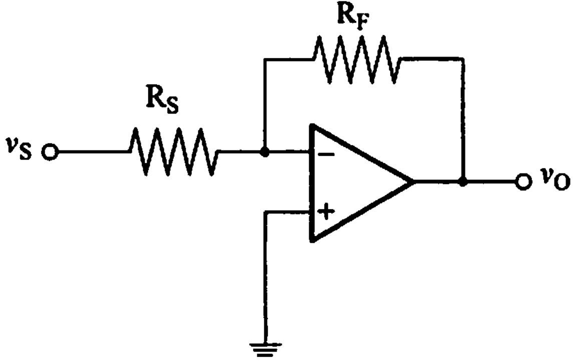


Ideal Operational Amplifier: When negative feedback is used, the input currents are zero ($i_+ = i_- = 0$) and the input differential voltage is zero $(v_+ - v_-) = 0$.

Some common operational amplifier circuits are shown below.

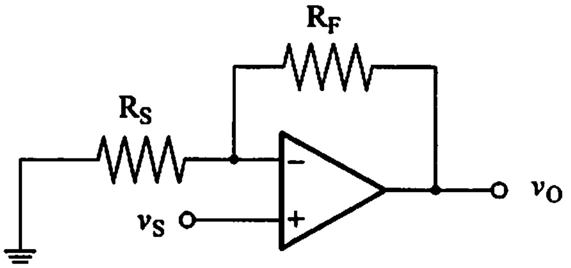
Inverting Amplifier

$A_v = v_o/v_s = -R_F/R_S$



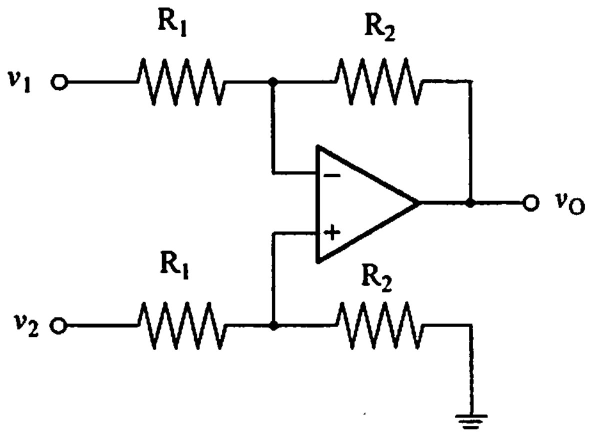
Non-Inverting Amplifier

$A_v = 1 + R_F/R_S$



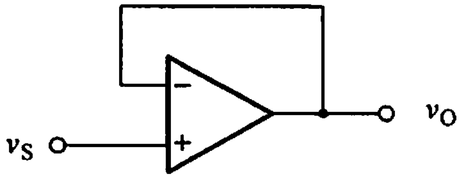
Difference Amplifier

$v_o = R_2/R_1(v_2 - v_1)$



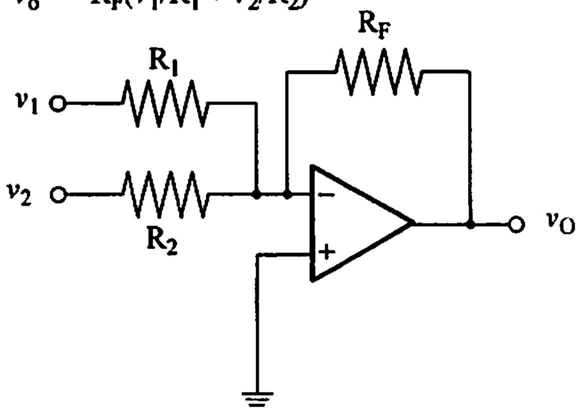
Voltage Follower

$v_o = v_s$



Summing Amplifier

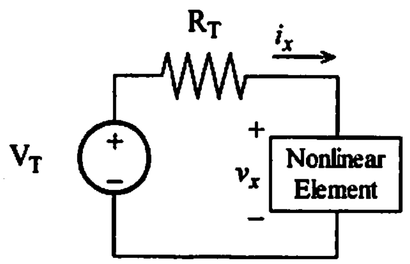
$v_o = -R_F(v_1/R_1 + v_2/R_2)$

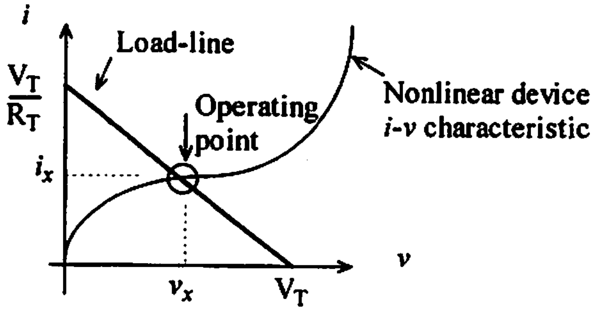


Discrete Electronic Devices

Electronic devices are nonlinear circuit elements. Circuits containing nonlinear circuit element are analyzed using two methods.

Load-Line Analysis:

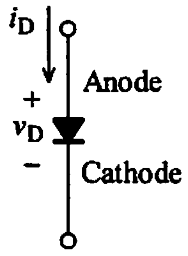




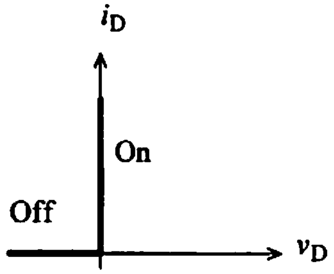
Piecewise Linear Models: With this technique the nonlinear characteristic is divided into linear segments. An assumption is made regarding the region of operation and then the validity of the assumption must be checked.

Semiconductor Diodes: A diode conducts readily in one direction but prevents the flow of current in the other direction. In other words a diode is an electric circuit element that functions like a one-way valve for current. Rectifier circuits are a common application of diodes.

Symbol

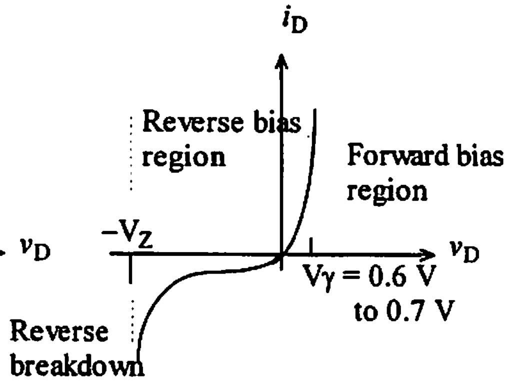


Ideal i-v characteristics



A diode that is On
is modeled as a short.
A diode that is Off
is modeled as an open.

Real i-v characteristics



Sample Questions of the Electrical Engineering Section of the FE Examination

C.1 Assuming the connecting wires and the battery have negligible resistance, the voltage across the 25-Ω resistance in Figure C.1 is

a. 25 V b. 60 V c. 50 V d. 15 V e. 12.5 V

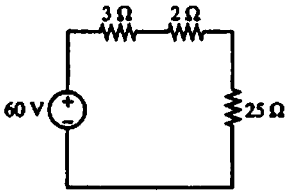


Figure C.1

C.2 Assuming the connecting wires and the battery have negligible resistance, the voltage across the 6-Ω resistor in Figure C.2 is

a. 6 V b. 3.5 V c. 12 V d. 8 V e. 3 V

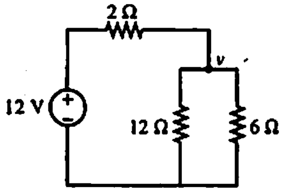


Figure C.2

C.3 A 125-V battery charger is used to charge a 75-V battery with internal resistance of 1.5 Ω. If the charging current is not to exceed 5 A, the minimum resistance in series with the charger must be

a. 10 Ω b. 5 Ω c. 38.5 Ω d. 41.5 Ω e. 8.5 Ω

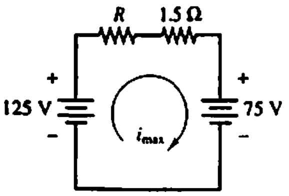


Figure C.3

C.4 A coil with inductance of 1 H and negligible resistance carries the current shown in Figure C.4. The maximum energy stored in the inductor is

a. 2 J b. 0.5 J c. 0.25 J d. 1 J e. 0.2 J

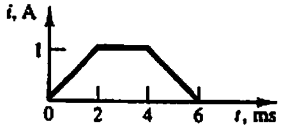


Figure C.4

C.5 The maximum voltage that will appear across the coil is

a. 5 V b. 100 V c. 250 V d. 500 V e. 5,000 V

C.6 A voltage sine wave of peak value 100 V is in phase with a current sine wave of peak value 4 A. When the phase angle is 60° later than a time at which the voltage and the current are both zero, the instantaneous power is most nearly

a. 250 W b. 200 W c. 400 W d. 150 W e. 100 W

C.7 A sinusoidal voltage whose amplitude is $20\sqrt{2}$ V is applied to a 5-Ω resistor. The root-mean-square value of the current is

a. 5.66 A b. 4 A c. 7.07 A d. 8 A e. 10 A

C.8 The magnitude of the steady-state root-mean-square voltage across the capacitor in the circuit of Figure C.5 is

a. 30 V b. 15 V c. 10 V d. 45 V e. 60 V

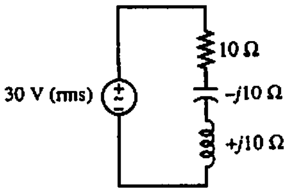


Figure C.5

The next set of questions (Exercises C.9 to C.28) pertain to single-phase AC power calculations and refer to the single-phase electrical network shown in Figure C.6. In this figure, $\mathbf{E}_S = 480\angle 0^\circ$ V; $\mathbf{I}_S = 100\angle -15^\circ$ A; $\omega = 120\pi$ rad/s. Further, load A is a bank of single-phase induction machines. The bank has an efficiency η of 80 percent, a power factor of 0.70 lagging, and a load of 20 hp. Load B is a bank of overexcited single-phase synchronous machines. The machines draw 15 kVA, and the load current leads the line voltage by 30°. Load C is a lighting (resistive) load and absorbs 10 kW. Load D is a proposed single-phase capacitor that will correct the source power factor to unity.

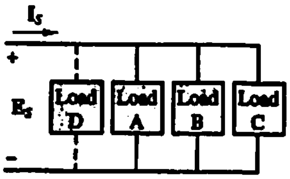


Figure C.6

C.9 The root-mean-square magnitude of load A current, denoted by I_A, is most nearly

a. 44.4 A b. 31.08 A c. 60 A d. 38.85 A e. 55.5 A

C.10 The phase angle of $\mathbf{I}_A$ with respect to the line voltage $\mathbf{E}_S$ is most nearly

a. 36.87° b. 60° c. 45.6° d. 30° e. 48°

C.11 The power absorbed by synchronous machines is most nearly

a. 20,000 W b. 7,500 W c. 13,000 W d. 12,990 W e. 15,000 W

C.12 The power factor of the system before load D is installed is most nearly

a. 0.70 lagging b. 0.866 leading c. 0.866 lagging
d. 0.966 leading e. 0.966 lagging

C.13 The capacitance of the capacitor that will give a unity power factor of the system is most nearly

a. 219 μF b. 187 μF c. 132.7 μF d. 240 μF e. 132.7 pF

C.14 The expression for the current in the 2-Ω resistor in Figure C.7 for time greater than zero is

a. $-3e^{-0.5t}+3$ A b. $3e^{-0.5t}+3$ A c. $-3e^{0.5t}+3$ A
d. $-6e^{0.5t}+6$ A e. $6e^{-0.5t}+6$ A

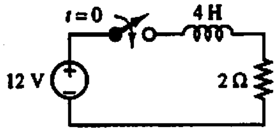


Figure C.7

C.15 A three-phase circuit is shown in Figure C.8. Load resistors (66 Ω) are connected in delta and supplied by a 220-V balanced three-phase source through three lines of 2-Ω resistance. The magnitude of the root-mean-square, line-to-line voltage across each 66-Ω resistor is most nearly

a. 198 V b. 110 V c. 201 V d. 220 V e. 120 V

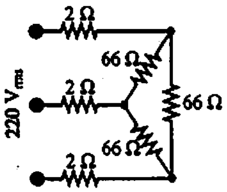


Figure C.8

C.16 A three-phase load is composed of three impedances of $9.0+j9.0$ Ω and connected in wye. The balanced three-phase source is 208 V (line to line). The current in each line is most nearly

a. 40 A b. 16.3 A c. 13.3 A d. 9 A e. 6 A

The next four exercises refer to a three-phase system with line-to-line voltage of 220 V rms, with ABC phase sequence and with phase reference V_{AB} shown in the phase diagram of Figure C.10. The load is a balanced delta connection, shown in Figure C.11 with branch impedances $Z = 30 - j40$ Ω, $j=\sqrt{-1}$.

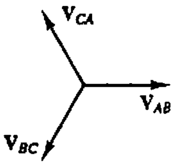


Figure C.10

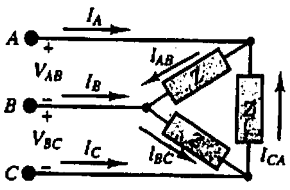


Figure C.11

C.17 The phase current is most nearly

a. $4.4\angle 53.13^\circ$ A b. $2.4\angle 53.13^\circ$ A c. $4.4\angle 0^\circ$ A
d. $4.4\angle -53.13^\circ$ A e. $2.4\angle -53.13^\circ$ A

C.18 The line current I_A (in amperes) is most nearly

a. $4.4\angle -186.87^\circ$ b. $4.4\angle 23^\circ$ c. 7 d. $7.6\angle 23^\circ$ e. $7\angle -186.87^\circ$

C.19 The power factor is most nearly

a. 1.0 b. 0.6 leading c. 0.866 leading d. 0 e. 0.8 lagging

C.20 The total real power delivered from the source to the load is most nearly

a. 1,496 W b. 580 W c. 1,742 W d. 2,904 W e. 850 W

C.21 The circuit of Figure C.13 is a

a. Peak detector b. Half-wave rectifier c. Bridge rectifier
d. Voltage doubler e. Full-wave rectifier

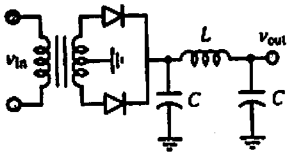


Figure C.13

C.22 The inductor L and the capacitor C serve the function of

a. Converting the AC input to DC output
b. Increasing the peak value of the output voltage
c. Protecting the diodes
d. A high-pass filter
e. Reducing the ripple component of the output voltage

C.23 The ideal diode D in Figure C.14 will always conduct if

a. V_1 is greater than V_2. b. V_2 is greater than V_1.
c. V_1 is greater than 1 V. d. R_2 is an open circuit.
e. R_1 is an open circuit.

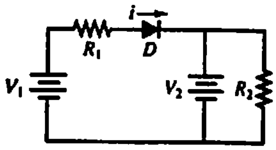


Figure C.14

C.24 In the circuit of Figure C.15, which value is closest to v_3 if $R_1 = 2.2\ \text{k}\Omega$, $R_2 = 1.5\ \text{k}\Omega$, $R_3 = 18\ \text{k}\Omega$, $v_1 = 120$ mV, and $v_2 = -40$ mV?

a. −250 mV b. 500 mV c. −500 mV d. 1.46 V e. −1.46 V

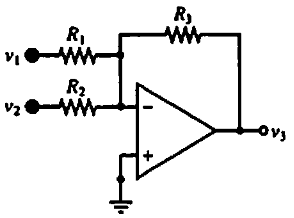


Figure C.15

C.25 In Figure C.15, if $R_1 = 2.2\ \text{k}\Omega$, $R_3 = 18\ \text{k}\Omega$, $v_1 = 120$ mV, and $v_2 = -40$ mV, choose the value of R_2 such that $v_3 = 0$.

a. 1.2 kΩ b. 5 kΩ c. 7.33 kΩ d. 0.733 kΩ e. 0.5 kΩ

C.26 Which of the following is a true characteristic of magnetic flux lines?

a. They cross each other.
b. They begin and end on electric charges.
c. They are parabolic.
d. They are continuous.
e. None of the above.

C.27 For the circuit of Figure C.16, where $i = 2$ A, $\phi = 1 \times 10^{-3}$ Wb, cross-sectional area $= 5\ \text{in}^2$, and the mean flux path length $= 2$ in, the total reluctance $\mathcal{R}$ of the magnetic circuit in (A-turns)(in^2)/Wb is

a. 1×10^5 b. 2×10^5 c. 1.5×10^5 d. 3.5×10^4 e. 2×10^5

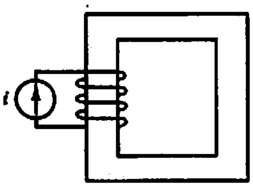


Figure C.16

Answers:

C.1: c;
C.2: d;
C.3: e;
C.4: b;
C.5: d;
C.6: e;
C.7: b;
C.8: a;
C.9: e;
C.10: c;
C.11: d;
C.12: e;
C.13: c;
C.14:?;
C.15: c;
C.16: d;
C.17: a
C.18: d;
C.19: b;
C.20: c;
C.21: e;
C.22: e;
C.23: a;
C.24: c;
C.25: d;
C.26: d;
C.27: b or e

Corrections:

Problem C.14: The correct answer is not one of the choices given!

$$i(t) = (6 - 6e^{-0.5t}) \text{ Amps}$$

Problem C.27: N = 100: To work this problem we need to know the number of turns in the coil.

Please note that two of the answers provided are correct (b or e)

PC1. A dc current of 3 A flows through an initially uncharged capacitor. After two microseconds, the magnitude of the net electric charge on one plate of the capacitor is most nearly:

a. 3 μC	**c.** 0 μC
b. 6 μC	**d.** 0.667 μC

PC2. For the circuit of Figure PC.2, the power dissipated in the 10-Ω resistor is most nearly:

a. 22.5 W

b. 8.27 W

c. 1.84 W

d. 7.35 W

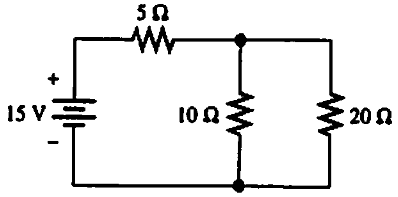


Figure PC.2

PC3. Two initially uncharged capacitors have values of 6 μF and 12 μF. The capacitors are connected in series, and a 200-V dc source is applied to the combination. The charge taken from the source is most nearly:

a. 800 μC

b. 3600 μC

c. 600 μC

d. 1200 μC

PC4. A 2-hp 220-V-rms single-phase induction motor operates at full load with 80% efficiency and 0.75 lagging power factor. The magnitude of the rms motor current is most nearly:

a. 4.07 A

b. 11.3 A

c. 6.35 A

d. 8.78 A

PC5. A 30-Ω resistor, a pure capacitance having a reactance magnitude of 80 Ω, and a pure inductance having a reactance magnitude of 40 Ω are in series. The impedance magnitude of the series combination is most nearly:

a. 150 Ω

b. 50 Ω

c. 14.1 Ω

d. 17. 1 Ω

PC6. The apparent power supplied to a load in an ac circuit is 2000 volt-amperes with a power factor of 0.6 lagging. The reactive power is most nearly:

a. 1200 VAR

b. 3333 VAR

c. 1600 VAR

d. 2500 VAR

PC7. A 150-microfarad capacitor has been charged to a potential of 100 V. A 50-Ω resistor is placed across the capacitor. After 20 time constants, the total energy delivered to the resistor is most nearly:

- **a.** 1.5 J
- **b.** 0 J
- **c.** 0.75 J
- **d.** 15×10^{-3} J

PC8. The current through the 50-Ω resistor for the circuit shown in Figure PC.8 is most nearly:

- **a.** 1.56 A rms
- **b.** 1.10 A rms
- **c.** 2.20 A rms
- **d.** 0.52 A rms

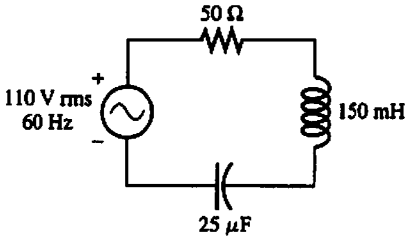


Figure PC.8

PC9. For time t greater than zero, the mathematical expression of the current through the 25-Ω resistance of Figure PC.9 is:

- **a.** $1 - \exp(-2t)$ A
- **b.** $1 - \exp(-t/2)$ A
- **c.** $\exp(-2t)$ A
- **d.** $\exp(-t/2)$ A

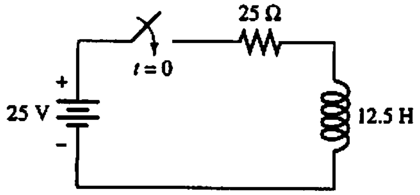


Figure PC.9

PC10. The output voltage v_o in the ideal op-amp circuit shown in Figure PC.10 is:

- **a.** $5v_1 + 2.5v_2$
- **b.** $10v_1 + 5v_2$
- **c.** $-5v_1 - 2.5v_2$
- **d.** $-10v_1 - 5v_2$

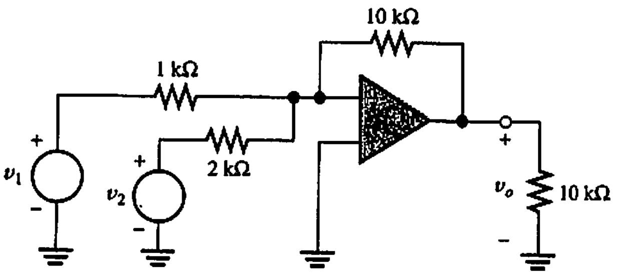


Figure PC.10

PC11. The output voltage v_o in the ideal op-amp circuit shown in Figure PC.11 is most nearly:

a. 4 V

b. 6 V

c. 2 V

d. 8 V

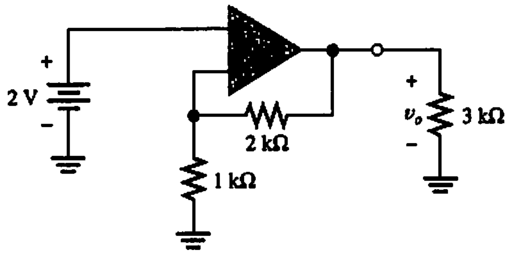


Figure PC.11

PC12. The voltage v_o shown in Figure PC.12 is best described as:

a. 21.2 V dc

b. 15 V dc

c. 15 V ac

d. a half-wave rectified sine wave

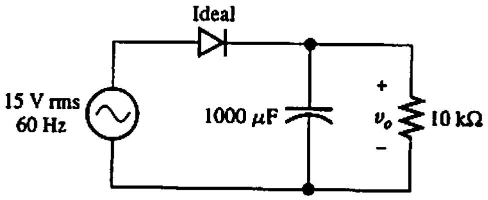


Figure PC.12

PC13. The voltage v_o shown in Figure PC.13 is most nearly:

a. 5 V

b. 7.5 V

c. 0 V

d. 10 V

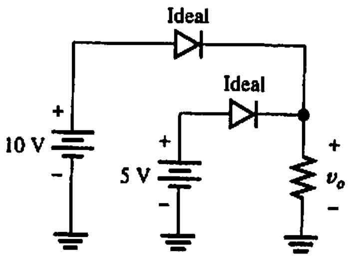


Figure PC.13

PC14. The current flowing through the 5-Ω resistor in Figure PC.14 is most nearly:

a. 2 A rms

b. 4 A rms

c. 1 A rms

d. 0 A rms

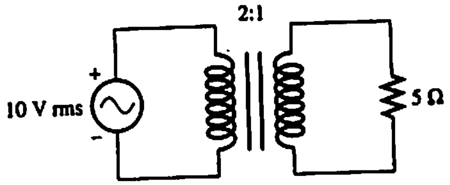


Figure PC.14

Answers:

PC1. b;
PC2. d;
PC3. a;
PC4. b;
PC5. b;
PC6. c;
PC7. c;
PC8. a;
PC9. a;
PC10. d;
PC11. b;
PC12. a;
PC13. d;
PC14. c;